Area-average solar radiative transfer in three-dimensionally inhomogeneous clouds: The Independently Scattering Cloudlets model

Grant W. Petty

Atmospheric and Oceanic Sciences, University of Wisconsin-Madison

Abstract.
A new conceptual and computational basis is described for renormalizing the single-scatter and extinction properties (optical depth, single-scatter albedo, and scattering phase function or asymmetry parameter) of a three-dimensionally inhomogeneous cloud volume or layer so as to describe a radiatively equivalent homogeneous volume or layer. The renormalization may allow area-averaged fluxes and intensities to be efficiently computed for some inhomogeneous cloud fields using standard homogeneous (e.g., plane-parallel) radiative transfer codes.

In the Independently Scattering Cloudlet (ISC) model, macroscopic “cloudlets” distributed randomly throughout a volume are treated as discrete scatterers, analogous to individual cloud droplets but with modified single-scatter properties due to internal multiple scattering. If a volume encompasses only cloudlets that are optically thin, the renormalized single scatter properties for the volume revert to the intrinsic values and the homogeneous case is recovered.

Although the ISC approach is based on a highly idealized, and therefore unrealistic, geometric model of inhomogeneous cloud structure, comparisons with accurate Monte Carlo flux calculations for more realistic random structures reveal surprising accuracy in its reproduction of the relationship between area-averaged albedo, direct transmittance, diffuse transmittance, and in-cloud absorptance. In particular, it describes the approximate functional dependence of these characteristics on the intrinsic single-scatter albedo when all other parameters are held constant. Moreover, it reproduces the relationship between renormalized single scatter albedo and renormalized optical thickness derived independently, via a perturbative analysis, by other authors. Finally, the ISC model offers a reasonably intuitive physical interpretation of how cloud inhomogeneities influence area-averaged solar radiative transfer, including the significant enhancement of in-cloud absorption under certain conditions.

1. Introduction
The modeling and parameterization of area-averaged radiative transfer in inhomogeneous cloud fields remains one of the most theoretically challenging problems in atmospheric radiation. The challenge is threefold: 1) that of representing the detailed microphysical and geometric structure of natural clouds in a statistically realistic way (Tao et al., 1987; Cahalan and Snider, 1989; Cahalan and Joseph, 1989; Joseph and Cahalan, 1990; Davis et al., 1993; Borde and Isaka, 1996), 2) the computational effort of performing accurate 3-D radiative transfer calculations for either a single realization or a stochastic model of a complex cloud field (Stephens, 1988; Davis et al., 1990, 1991; Audic and Frisch, 1993; Evans, 1993; Malvagi et al., 1993; Peltoniemi, 1993; Zuev and Titov, 1995; Davis and Marshak, 1997; Galin- sky and Ramanathan, 1998; Galinsky, 2000) and, especially, 3) the question of how to generalize from a plethora of specific results to a few universally applicable principles governing area-average radiative transfer in naturally occurring cloud fields (Stephens, 1988; Titov, 1990; Gabriel and Evans, 1996; Davis and Marshak, 2001). The last problem is of particular importance for radiative parameterizations utilized in climate models and numerical weather prediction models (Stephens, 1984; Schmetz, 1984; Harshwardhan and Espinoza, 1995; Barker, 1996).

One common approach to the latter problem is to assume that subgrid-scale cloudiness can be treated locally as plane-parallel, so that grid-averaged solar fluxes depend only on the probability distribution of cloud optical depths. This so-called Independent Pixel Approximation (IPA) has been shown to be reasonably accurate...
for at least some types of stratiform clouds (Cahalan et al., 1994; Barker, 1996; Barker et al., 1996; Zuidema and Evans, 1998), and methods for parameterizing the IPA in large-scale radiative transfer calculations have been developed by Barker (1996) and Oreopoulos and Barker (1999).

The IPA seems likely to fail, however, in broken cloud fields in which individual cloud elements are not well-approximated as plane-parallel (e.g., Fig. 1). Furthermore, it is reasonable to conjecture that, even in contiguous cloud layers, three-dimensional internal inhomogeneities in cloud density may potentially modify the area-averaged radiative properties of the clouds in a manner that cannot be adequately accounted for by the IPA.

In addition to the practical problem of parameterizing radiation in models, potentially important theoretical questions are raised by the so-called cloud absorption anomaly discussed by Stephens and Tsay (1990); Cess et al. (1995); Francis et al. (1997) and many others. The radiative effects of internal inhomogeneities in clouds have not been ruled out as one possible contributor to enhanced absorption (Kliorin et al., 1990; Li et al., 1995; Byrne et al., 1996), though other authors have found little evidence of such an effect (Barker et al., 1998).

Furthermore, remote sensing at solar wavelengths has become an important means for collecting information global cloud properties (Rossow and Schiffer, 1991), yet traditional remote sensing methods have assumed that the clouds are plane-parallel on the scales of individual pixels. There is growing evidence that this assumption is inappropriate in at least some contexts (Davies, 1984; Marshak et al., 1995; Loeb et al., 1997; Loeb and Coakley, 1998), especially with respect to the angular distribution of reflected intensities. For such cases, new retrieval methods are needed that can explicitly account for possible 3-D inhomogeneity within a satellite pixel.

Above all, there is a need for a clear conceptual model of just how inhomogeneities in cloud fields influence solar radiative transfer. Theoretical treatments to date of 3-D inhomogeneity have tended to rely on mathematical methods that convey little physical insight to most non-specialists.

This paper thus has two specific objectives, one primarily academic, and the other practical. The first is to propose a new conceptual basis — the Independently Scattering Cloudlet (ISC) model — for visualizing and explaining the effects of random 3-D inhomogeneity on area- or volume-averaged radiative transfer. Second, it is demonstrated empirically that, despite its rather drastic geometric assumptions, the model nevertheless appears to offer a credible quantitative basis for renormalizing the optical properties of quasi-realistic inhomogeneous cloud structures so as to permit efficient flux calculations using standard homogeneous radiative transfer codes.

The second of the above two objectives, that of recasting an inhomogeneous cloud layer or volume as a radiatively equivalent homogeneous layer or volume, is identical to that of Cairns et al. (2000), as well as that of previous authors for more restrictive models of clouds or other “clumpy” scattering media (Boisse, 1990; Kliorin et al., 1990; Malvagi and Pomraning, 1990; Hobson and Scheuer, 1993; Hobson and Padman, 1993; Városi and Dwek, 1999). Indeed, over a broad range of the relevant inhomogeneity parameter, the ISC results are in near-perfect agreement with Cairns’ formulation for the renormalized single-scatter albedo. However, the present derivation follows an entirely different line of physical reasoning and it appears, based on preliminary empirical evidence, to be less restricted in its range of validity. Among other things, the ISC model avoids the need to artificially distinguish between small-scale fluctuations in cloud water density, which should have no significant radiative effect, and those fluctuations whose dimensions are comparable to or larger than the photon mean free path. Indeed, neither the geometric length scale of variability nor the statistical distribution of point-wise cloud water density play any direct role in the ISC model; rather, our analysis yields relationships that depend solely on a simple partitioning of total cloud water between “homogeneous” and “inhomogeneous” components and on an effective mean optical
thickness $\tau'$ of the inhomogeneities.

The next section reviews the computation of radiative transfer properties from dropsize distributions (DSDs) for homogeneous clouds and then lays the conceptual foundation for a generalization of this approach to clouds containing macroscopic scattering elements, or “cloudlets.” The effective single-scatter properties of these cloudlets are related to those of the underlying DSD but are modified to account for multiple scattering.

Section 3 undertakes a detailed analysis of the effective single-scatter properties of individual cloudlets, under the assumption that they are both spherical and internally homogeneous. Section 4 then describes, and validates, the use of a conventional plane-parallel RT code to compute domain-averaged fluxes for a hypothetical cloud layer composed of a random distribution in 3-D space of identical macroscopic cloudlets separated by clear air. In addition, Section 4 discusses the general predictions of the ISC model for cloud albedo, direct and diffuse transmittance, and in-cloud absorption. Among other things, it confirms that internal inhomogeneities alone are capable of significantly enhancing absorption in an optically thick cloud layer. Section 5 discusses the striking similarities, and notable differences, in the predictions of the single-parameter ISC model and the renormalization method method of Cairns et al. (2000).

Section 6 extends the one-parameter description of inhomogeneous clouds, one based strictly on the characteristic optical dimension $\tau'$ of the internal inhomogeneities, to a two-parameter model of inhomogeneities that includes an arbitrary component of homogeneous cloud, within which the inhomogeneous elements may be embedded. It is argued heuristically, in part via an analysis based on similarity theory, that the two-parameter version of the model is both necessary and sufficient to reproduce all physically admissible combinations of area-average in-cloud absorption, direct and diffuse transmittance, and albedo predicted by the ISC model.

Finally, in Section 7, it is shown empirically, through comparison with accurate Monte Carlo calculations, that the two-parameter ISC model correctly describes the functional relationships between intrinsic single-scatter albedo, area-averaged albedo, transmittance, absorption, etc., for simulated random 3-D cloud structures not satisfying the highly idealized assumptions used in the original derivation. In light of this finding, it is suggested that the ISC model may have at least some relevance to the parameterization of solar radiative transfer in quasi-realistic chaotic cloud fields, as well as providing new physical insights into the nature of radiative transfer in such clouds. The question addressed most directly in this initial study, however, is not whether the ISC model has any verifiable applicability to real clouds, but rather whether it is valid on its own terms; i.e., as a remarkably simple yet accurate model of stochastic radiative transfer in at least some classes of randomly inhomogeneous scattering media.

2. Model Development

Within any extended volume of cloudy air, subregions may be identified that are optically denser, on average, than their immediate surroundings. The fundamental premise of the ISC model developed herein is that the combined radiative effects of these structures within an extended volume of cloudy area may be approximated by treating them as macroscopic discrete scattering entities, whose effective “single-scatter” properties are re-
G.W. Petty: Radiative transfer in inhomogeneous clouds

Fig. 3. Schematic illustrating the role of optical thickness on the effective scattering properties of a finite cloudlet. a) An optically thin cloudlet \( \tau' \ll 1 \). b) An optically thick cloudlet \( \tau' \gg 1 \).

An extended volume of inhomogeneously cloudy air is therefore modeled as a three-dimensional random ensemble of independently scattering cloudlets widely separated by clear air (Fig. 2). The latter condition implies that only the overall angular distribution of scattered radiation from each cloudlet need be considered. It is further assumed that the spatial positions of these elements are statistically independent of one another and that the statistical distribution of cloudiness along any given ray within the extended volume is directionally isotropic.

Under these admittedly restrictive assumptions, one may evaluate the domain-averaged radiative properties of the inhomogeneous cloud volume in a manner completely analogous to homogeneous clouds containing a prescribed size distribution of individual cloud droplets, as summarized in the following subsection. The understandable objection that such an inhomogeneous medium bears little physical resemblance to any real cloud may be deflected for now by regarding it as a mere thought experiment. Later, however, it will be shown through empirical comparison with Monte Carlo calculations that the model indeed seems to reproduce the essential radiative properties of a far less restrictive class of inhomogeneous cloud structures.

a. Intrinsic optical properties of cloud water

Conventionally, the scattering and extinction characteristics of a small volume within a cloud composed of spherical water droplets are determined by first calculating the Mie extinction and scattering properties of individual droplets as functions of droplet radius \( r \) and then integrating the results over an appropriate drop size distribution \( n(r) \) to obtain the total volume extinction coefficient \( k \), the single scatter albedo \( \omega_0 \), and a scattering phase function \( P(\Theta) \), where \( \Theta \) is the angle of scattering relative to the incident radiation:

\[
k = \int_0^{\infty} C_{\text{ext}}(r)n(r) \, dr, \quad (1)
\]

\[
\omega_0 = \frac{1}{k} \int_0^{\infty} C_{\text{sca}}(r)n(r) \, dr, \quad (2)
\]

\[
P(\Theta) = \frac{1}{\omega_0 k} \int_0^{\infty} C_{\text{sca}}(r)P(\Theta; r)n(r) \, dr, \quad (3)
\]

where

\[
C_{\text{ext}} = \pi r^2 Q_{\text{ext}}, \quad C_{\text{sca}} = \pi r^2 Q_{\text{sca}}, \quad (4)
\]

and \( Q_{\text{ext}} \) and \( Q_{\text{sca}} \) are the extinction and scattering efficiencies, respectively.

Recall that \( \omega_0 \) and \( P(\Theta) \) depend only on the relative numbers of drops of different radii \( r \), whereas \( k \) depends on their absolute number concentration as well. For the purposes of the analysis to follow, it is convenient to instead focus on the mass extinction coefficient \( \sigma \) (extinction cross-section per unit mass of cloud water), whose value is invariant with respect to cloud liquid water density \( w \), provided only that the shape of \( n(r) \) does not vary:

\[
\sigma \equiv \frac{k}{w}, \quad (5)
\]

where

\[
w = \rho \int_0^{\infty} \frac{4}{3} \pi r^3 n(r) \, dr, \quad (6)
\]
and $\rho_l$ is the density of pure liquid water.

For illustration purposes, we may assume that $Q_{ext} \approx 2$, in which case

$$\sigma \approx \frac{3}{2 \rho_l r_e},$$

(7)

where $\rho_l$ is the density of pure liquid water. If the effective droplet radius $r_e$ is taken to be $10 \ \mu m$, then $\sigma \approx 150 \ m^2 \ kg^{-1}$.

In summary, for any cloud whose liquid water density $w$ is known and constant throughout a finite volume $V$, its radiative properties within that volume are completely and uniquely determined by three parameters: the mass extinction coefficient $\sigma$, the single scatter albedo $\bar{\omega}_0$, and the scattering phase function $P(\Theta)$. For many purposes, such as flux calculations, the asymmetry factor $g$ is sufficient to characterize $P(\Theta)$.

As a special case, the radiative properties of a homogeneous plane-parallel cloud with total column liquid water $W$ are determined by $\bar{\omega}_0$, $P(\Theta)$ (or $g$), and by the total optical thickness $\tau^*$, which is given by

$$\tau^* = \sigma W$$

(8)

The optical thickness $\tau^*$ in turn is uniquely related to the direct transmittance of solar radiation according to Bouger-Lambert-Beer’s Law (henceforth simply “Beer’s Law”):

$$T_{dir} = \exp \left[ -\frac{\tau^*}{\mu_0} \right]$$

(9)

where $\mu_0$ is the cosine of the solar zenith angle.

The objective of this paper is to replace the above intrinsic optical parameters with analogous effective parameters which account for random fluctuations of cloud water density within the volume $V$.

Conceptually at least, the motivation behind renormalization is most easily understood in the case of the adjusted optical thickness $\tau^*_{eff}$. If this parameter is to have the intended meaning in the context of the present development, its definition must follow from Beer’s Law; thus, it is uniquely determined by the horizontally averaged direct transmittance $T_{dir}$, including the contributions of any breaks and/or thin spots in the layer:

$$\tau^*_{eff} \equiv -\mu_0 \log \left[ T_{dir} \right]$$

(10)

As an example of the application of (10), consider the cloud field depicted in Fig. 1. In this case, subjective observation by the author of the pattern of shadows on the ground beneath the cloud layer (not shown) suggested an area-averaged direct transmittance of approximately 20%. For a solar zenith angle of approximately $45^\circ$, this observation implies a value for $\tau^*_{eff}$ of $\sim 1$. On the other hand, in view of typical cumulus cloud water contents, the area-averaged liquid water path $W$ of the scene was presumably on the order of $10^{-1} \ kg/m^2$. Thus, $\tau^*_{eff}$ is an order of magnitude or more smaller than the estimated area-averaged optical depth $\tau^* \sim 15$.

As discussed by Cairns et al. (2000), not only $\tau^*$ but also the other radiative properties, i.e., single-scatter albedo and scattering phase function, must be renormalized to account for internal inhomogeneities within a cloud volume. Moreover, it is essential that these properties be adjusted in a physically self-consistent way. Although one may adjust $\tau^*$ by itself to achieve the correct area-averaged cloud-top albedo, this approach will not, in general, yield correct values for the transmitted fluxes (diffuse or direct) or in-cloud absorption. The ISC model proposed in this paper provides one mechanism for obtaining physically self-consistent adjustments of optical thickness, single scatter albedo, and scattering phase function so as to simultaneously obtain the correct area-average albedo, diffuse transmittance, direct transmittance, and absorptance. More importantly, it provides a physical basis for extrapolating these adjustments across various values of the intrinsic single scatter albedo while keeping the cloud structure constant, as required for example in remote sensing applications or in the calculation of broadband fluxes.

b. The Independently Scattering Cloudlet Model

Consider a finite volume $V$ of cloud containing a random ensemble of $i$ non-overlapping, radially symmetric cloudlets, each with its own uniform liquid water density $w_i$. It is convenient, though not necessary, to model these as homogeneous spheres with geometric radius $R_i$. The total liquid water mass contributed by the $i$th cloudlet is then

$$M_i = \frac{4}{3} \pi R_i^3 w_i.$$  

(11)

Analogous to cloud droplets, each cloudlet may then be treated as an independent scatterer with its own extinction cross-section $C_{ext,i}$, single scatter albedo $\bar{\omega}_{0,i}$, and scattering phase function $P_i(\Theta)$. If one further assumes that the cloud liquid water in all cloudlets possess the same intrinsic optical properties $\sigma$, $\bar{\omega}_0$, and $P(\Theta)$, as computed from (2)–(6), then $w_i$ and $R_i$ uniquely determine all far-field scattering parameters of the entire cloudlet viewed as a single “particle.” (Hereafter, unprimed optical parameters refer to intrinsic values, single-primed parameters refer to the effective optical properties of individual cloudlets, and double-primed
Summing appropriately over the distribution of cloudlets yields the mean liquid water content \( \bar{\varpi} \) for volume \( V \) and effective optical parameters \( k'' \), \( \sigma'' \), \( \varpi_0'' \), \( P''(\Theta) \), and \( g'' \):

\[
\bar{\varpi} = \frac{1}{V} \sum_i M_i \tag{12}
\]

\[
k'' = \frac{1}{V} \sum_i C'_{ext,i} \tag{13}
\]

\[
\sigma'' = \frac{k''}{\bar{\varpi}} \tag{14}
\]

\[
\varpi_0'' = \frac{1}{Vk''} \sum_i C'_{ext,i} \varpi_0' \tag{15}
\]

\[
P''(\Theta) = \frac{1}{Vk''} \sum_i C'_{ext,i} \varpi_0' P'_i(\Theta) \tag{16}
\]

\[
g'' = \frac{1}{Vk''} \sum_i C'_{ext,i} \varpi_0' g_i \tag{17}
\]

Equations (11)–(17) represent the formal basis for recasting an inhomogeneous cloud as one which is homogeneous throughout volume \( V \). The next sections address the problem of obtaining \( C'_{ext} \), \( \varpi_0' \), and \( P'(\Theta) \) for individual cloudlets.

3. Single-scattering properties of cloudlets

Let the intrinsic optical properties \( \sigma \), \( \varpi_0 \), and \( P(\Theta) \) of the cloud liquid water contained in a homogeneous sphere of radius \( R \) be fixed. The effective properties \( \sigma' \), \( \varpi_0' \), and \( P'(\Theta) \) of that sphere depend only on the non-dimensional optical diameter \( \tau' \) of the sphere, which we define as:

\[
\tau' \equiv 2kR = 2\sigma w R = \frac{3\sigma M}{2\pi R^2} \tag{18}
\]

Any differences between \( \sigma \) and \( \sigma' \), \( \varpi_0 \) and \( \varpi_0' \), etc., are due to a combination of two processes — non-negligible attenuation of the incident beam of light as it passes into the interior of the sphere and multiple scattering within the sphere (Fig. 3). The value of \( \tau' \) will be shown below to have the following qualitative effect on the bulk radiative properties of a cloud volume:

- In the limit as \( \tau' \) goes to zero, the values of the primed quantities revert to the values of the corresponding unprimed (intrinsic) quantities, because both attenuation and multiple scattering within the cloudlet become negligible.
- In the limit as \( \tau' \) goes to infinity, the extinction cross-section of the cloudlet asymptotically approaches the geometric cross-section \( \pi R^2 \) and thus, from (18), \( \sigma' \to 0 \). In general, \( \sigma' < \sigma \) for \( \tau' > 0 \).
- For \( \tau' > 0 \) and \( \varpi_0 < 1 \), the effective single scatter albedo \( \varpi_0' = \varpi_0'' \), where \( n \) is the mean number of internal scatterings before an incident photon exits the sphere. Therefore \( \varpi_0' < \varpi_0 \).
- For \( \tau' > 0 \) and \( \varpi_0 > 0 \), the effective asymmetry factor \( g' \) < \( g \), because multiple scattering reduces the fraction of incident photons which penetrate the cloudlet in the forward direction while increasing the fraction which exit the sphere in the backward direction.

a. Renormalized mass extinction coefficient

An analytic expression for the extinction cross section \( C'_{ext} \) of a cloudlet is derived by noting that the total radiation extinguished by the cloudlet is one minus the direct transmittance parallel to the incident beam and integrated over the cross-sectional area of the sphere:

\[
C'_{ext} = \int_0^R 2\pi r \left[ 1 - \exp\left( -2k \left( R^2 - r^2 \right) \right) \right] dr, \tag{19}
\]

which has the analytic solution

\[
C'_{ext} = \frac{\pi}{2k^2} \left[ 2k^2 R^2 + (2kR + 1)e^{-2kR} - 1 \right]. \tag{20}
\]

Dividing the above by \( M = (4/3)\pi R^3 w \) and using the definition of \( \tau' \) yields the renormalized mass extinction coefficient

\[
\sigma' = \phi(\tau')\sigma \tag{21}
\]

where the optical depth reduction factor is given by

\[
\phi(\tau') = \frac{6 \left( \tau' + 1 \right)e^{-\tau'} - 1}{2\tau'^3}. \tag{22}
\]

Since for small \( \tau' \) the above expression involves a delicate cancellation of terms and is therefore susceptible to roundoff error, the following asymptotic expression may be useful, which is accurate to better than 1% for \( \tau' < 0.5 \):

\[
\phi(\tau') \approx 1 - \frac{3}{8}\tau' + \frac{1}{8}\tau'^2. \tag{23}
\]

To similar accuracy, (22) reduces to

\[
\phi(\tau') \approx \frac{3\tau'^2 - 6}{2\tau'^3} \tag{24}
\]

for \( \tau' > 5 \).
The reduction in effective mass extinction coefficient (or optical depth), relative to the homogeneous (intrinsic) value, that results from treating a spherical "cloudlet" with optical thickness $\tau'$ as a discrete scattering entity. The solid curve is the untransformed reduction $\phi(\tau')$ factor calculated from (22). The dashed curve is the similarity-transformed reduction factor which accounts for the combined effect of $\phi(\tau')$ and the effective asymmetry factor $g'(\tau')$ on cloud albedo.

Figure (4) depicts the dependence of $\phi$ on $\tau'$. It is apparent that a cloud layer composed of discrete cloudlets, each having optical diameter $\tau' \sim 1$, will exhibit a significantly reduced optical depth $\tau_{\text{eff}}$ relative to a homogeneous cloud having the same mean liquid water path (note that $\tau' \sim 1$ implies geometric path lengths of order $10^{-10}$ m for typical liquid water densities encountered in clouds). This is of course the expected result: recalling that $\tau_{\text{eff}}$ is linked to the direct transmittance by Beer's Law (10), we have simply confirmed that inhomogeneous (e.g. broken) cloud with a given area-averaged liquid water content transmits more direct solar radiation to the surface than a homogeneous cloud with the same liquid water content.

Although similar closed-form expressions for the dependence of $\omega_0'$, $P'(\Theta)$, and $g'$ on cloudlet parameters may exist, their derivation would effectively require an analytic solution to the multiple scattering problem for a spherical cloud (see, however, Section 5, which discusses the applicability of the renormalization formulas of Cairns et al. 2000). The required functional relationships are therefore determined here numerically using a simple Monte Carlo code written for this purpose. Once the functional relationships have been tabulated for a given "family" of cloudlets having specified intrinsic $\omega_0$ and $P'(\Theta)$, they need never be recalculated again.

For illustration purposes, a representative range of single-scattering properties were computed for a cloudlet whose intrinsic scattering phase function $P'(\Theta)$ is the Henyey-Greenstein function with $g = 0.86$. Four values for the intrinsic single scatter albedo $\omega_0$ were assumed: 1.0, 0.999, 0.99, and 0.9. A total of 19 values of $\tau'$ were used, varying from 0.001 to 262.14 by powers of two. In order to examine the sensitivity of the results to $g$, a second set of calculations were performed for an isotropic phase function ($g = 0$).

b. Renormalized single scatter albedo $\omega_0'$

When $\tau' \gtrsim 1$, multiple scattering within each cloudlet becomes important. The effective (renormalized) single scatter albedo may be expressed as

$$\omega_0' = \omega_0^n,$$

where this relationship defines an effective mean number of internal scatterings $n$ before an incident photon exits the cloudlet. Thus, $\omega_0' \leq \omega_0$. Figure 5 shows the dependence of $\omega_0'$ on $\tau'$ for various values of $g$ and $\omega_0$. Figure 6 depicts $n$ as a function of $\tau'$ for weakly absorbing cases ($1 - \omega_0 < 0.01$) with $\tau'$ up to $\sim 10$, $n$ is seen to be nearly independent of either $g$ or $\omega_0$. 

![Optical depth renormalization factor](image1)

![Renormalized Single-Scatter Albedo](image2)
c. Renormalized asymmetry factor \( g' \)

Figure 7 depicts the dependence of the effective scattering asymmetry factor \( g' \) on \( \tau' \) for various values of \( \varpi_0 \) and \( g \). For \( \tau' < 1 \), there is little or no difference between \( g \) and \( g' \). For optically thicker cloudlets, there is the expected tendency toward more negative \( g' \) as multiple scattering enhances the intensity of radiation emerging from the incident side of the cloudlet while attenuating radiation in the forward direction. When \( g = 0.86 \), an optical thickness \( \tau' \approx 30 \) is sufficient to reduce \( g' \) to approximately zero. However, when absorption is strong (e.g., \( \varpi_0 = 0.9 \)), higher-order scattering is suppressed and the reduction in \( g' \) is therefore not as great.

4. Inhomogeneous layer clouds composed of identical cloudlets

Having determined the effective scattering properties of single cloudlets, we turn to the radiative properties of an extended cloud element or layer comprised of a random distribution in space of such cloudlets. The simplest such system is a plane parallel cloud layer with fixed average liquid water path \( \mathcal{W} \) and composed of identical cloudlets, each having the same optical thickness \( \tau' \) and intrinsic optical properties \( \sigma, \varpi_0, P'(\Theta) \). The effective optical depth of the layer is then simply

\[
\tau_{\text{eff}} = \phi(\tau') \sigma \mathcal{W}
\]

where \( \mathcal{W} = MN \),

and \( N \) is the number density (per unit area) of the cloudlets.

Area-averaged radiative transfer calculations may then be performed for this cloud layer using standard plane-parallel codes by substituting \( \tau_{\text{eff}} = \phi(\tau') \tau^*, \varpi_0'(\tau') \), and \( P'(\Theta; \tau') \) for their homogeneous counterparts. We have thus developed the important ability to transform a highly inhomogeneous cloud layer, whose internal structure is defined by the single parameter \( \tau' \), into a radiatively equivalent homogeneous cloud.

a. Validation of ISC Concept

The validity of this approach may be demonstrated empirically via direct comparison with full 3-D Monte Carlo radiative transfer calculations. For this example, a 3-D model domain was partially filled with identical homogeneous cloudy cubes having optical diameter \( \tau' = 8 \). The horizontal dimensions of the domain were \( 32 \times 32 \) with periodic lateral boundary conditions, and the vertical dimension was 64. One-eighth of the available grid boxes in the 3-D domain were randomly filled with cloud, for a horizontally averaged optical depth \( \tau^* = 64 \). Cloudy cubes were allowed to be contiguous. A Heneyy-Greenstein phase function was utilized with asymmetry parameter \( g = 0.86 \). The model domain was illuminated...
from above with a solar incidence angle of \( \theta_0 = 60^\circ \). The lower boundary was taken to be black. The Monte Carlo calculation utilized 10^6 photons for each instance of \( \omega_0 \), which ranged from 1 (conservative scattering) to 0.9 (strongly absorbing). Each simulation required up to 20 minutes on an HP C180 workstation. The simulations yielded horizontally averaged reflectivity, diffuse and direct transmittance, and in-cloud absorptance.

The comparable calculation utilizing the ISC model is undertaken by performing a plane-parallel radiative transfer calculation for the radiatively equivalent homogeneous cloud layer. For this example, the discrete ordinates radiative transfer code DISORT v1.1 (Stamnes et al., 1988) was employed. The above value of \( \tau' = 8 \) was utilized to obtain a renormalized optical depth \( \tau'_{\text{eff}} = 11.6 \) from (22), and renormalized single-scatter albedo \( \omega_0' \) and phase function \( P'(\Theta) \), as depicted in Figs. 5 and 7. The empirically derived phase function was then approximated as a series of Legendre polynomial coefficients, as required for input to DISORT. Sixteen streams were utilized in the DISORT calculation. The radiative transfer calculation required less than 4 seconds of CPU time, or more than two orders of magnitude less time than for the MC calculations. Results of the MC and discrete ordinates computations are presented in Table 1.

The agreement is outstanding, as all computed values are essentially indistinguishable between the two methods. Note that both the MC and ISC results depart substantially from those computed for a homogeneous plane-parallel cloud with otherwise identical properties. In particular, the inhomogeneous results show lower reflectivity, larger diffuse transmittance, and up to 15% greater in-cloud absorption.

Such agreement is especially noteworthy in view of the following deviations of the MC simulations from the idealized geometry underlying the ISC model derivation:

- The explicit MC simulations were performed on a domain occupied by cubical, rather than spherical, cloudlets, contrary to the assumption utilized to derive renormalized radiative properties in the previous section; and
- The cubical cloudlets in this simulation were fairly densely packed, each one being directly contiguous (sharing a planar boundary) with an average of 0.75 (= 6/8) other cloudlets and sharing a corner point with an average of one (= 8/8) additional cloudlet. Strictly speaking, the relatively frequent occurrence of such close contact between cloudlets violates a primary assumption underlying equations (14) through (17).

Thus, we not only have strong empirical validation of the conceptual basis of the renormalization, but also evidence that the numerical results are rather insensitive to deviations from the precise geometric assumptions utilized in developing the ISC model.

b. Limiting behavior for small \( \tau' \)

If \( \tau' \) is sufficiently small, then all optical properties revert to those of a homogeneous cloud having the same liquid water path. A cloud composed of discrete cloudlets which are all optically thin and which may even be widely separated by clear air is radiatively indistinguishable from a cloud which has the same average liquid water path \( \overline{W} \) but is perfectly homogeneous. Thus, the inhomogeneous ISC model has the desirable property of correctly modeling a homogeneous cloud as a special case.

Moreover, from (11), (12), and (18), it can be seen that there is a surprising variety of distinct ways to configure an inhomogeneous cloud layer with fixed \( \overline{W} \) so that it behaves radiatively as though it were homogeneous.

- For fixed \( R \), allow \( w \), and thus \( M \), to go to zero while \( N \to \infty \). This corresponds to the superposition of an infinite number of infinitely tenuous cloudlets of finite radius \( R \).
- For fixed \( M \) and \( N \), allow \( R \to \infty \), corresponding to the superposition of a finite number of large but tenuous cloudlets.
- For fixed \( w \), allow \( R \to 0 \) and \( N \to \infty \). This corresponds to the superposition of a large number of small cloudlets.

We have not been concerned about the conceptual problem of cloudlet overlap in the first two of the above three limiting cases. This is in part because the geometric depth of a plane parallel cloud layer is irrelevant to its radiative properties (since gaseous absorption is not being considered). One may therefore vertically stretch out the ensemble of cloudlets by as much as necessary (while conserving vertically integrated water mass) to avoid violating the geometric assumption of no overlap.

c. Analytically derived cloud layer properties

It is instructive to begin examining the qualitative effects of internal inhomogeneities on the area-averaged
Table 1. Comparison between “Independently Scattering Cloudlet” (ISC) model and direct Monte Carlo (MC) calculations for a 3-D random distribution of homogeneous scattering cubes with optical diameter $\tau' = 8$. Ratio of cloudy to clear air within Monte Carlo model domain is 1:8. For reference, results are also given for a plane-parallel (PP) cloud having the same mean optical thickness $\tau^*$. 

<table>
<thead>
<tr>
<th>$\omega_0$</th>
<th>$\omega_0'$</th>
<th>$g'$</th>
<th>Reflect.</th>
<th>Diff. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.466</td>
<td>ISC</td>
<td>0.845</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.844</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PP</td>
<td>0.887</td>
<td>0.113</td>
</tr>
<tr>
<td>0.999</td>
<td>0.994</td>
<td>0.471</td>
<td>ISC</td>
<td>0.781</td>
<td>0.119</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.781</td>
<td>0.119</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PP</td>
<td>0.828</td>
<td>0.079</td>
</tr>
<tr>
<td>0.99</td>
<td>0.946</td>
<td>0.474</td>
<td>ISC</td>
<td>0.518</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.516</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PP</td>
<td>0.587</td>
<td>0.009</td>
</tr>
<tr>
<td>0.9</td>
<td>0.589</td>
<td>0.516</td>
<td>ISC</td>
<td>0.125</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.126</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PP</td>
<td>0.198</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The contribution of decreasing $g'(\tau')$ to an increase in the similarity-transformed optical thickness. This function, multiplied by $\phi(\tau')$, yields the net dependence of the similarity-transformed effective optical thickness on $\tau'$, as depicted by the dashed curve in Fig. 4.
cloud with the same effective optical depth. Recall however that an increase in \( \tau' \) also reduces the effective mass extinction coefficient. The simultaneous decrease in both \( \tau^*_{\text{eff}} \) and \( g' \) due to inhomogeneity must be considered together in order to determine its net effect on the similarity-transformed optical depth. The combined effect is expressed by \( \left[(1 - g')/(1 - g)\right] \phi(\tau') \), which is plotted as a dashed curve in Fig. 4. Interestingly, the increased backscattering due to inhomogeneity delays by about one decade the effects of inhomogeneity on the similarity-transformed effective optical thickness. Albedo reductions due to inhomogeneity should therefore become significant only when \( \tau' \) is of order unity or greater, despite a significant reduction in effective optical depth for \( \tau' \) as small as \( 10^{-1} \). Note that in order to be consistent with the increase in direct transmittance occasioned by an increase in \( \tau' \), constant albedo requires an exactly compensating reduction in the diffuse transmittance for \( \tau' \) up to order unity. Thus it may be concluded that up to this value, inhomogeneity has no significant net effect except to alter the ratio of direct to diffuse transmittance.

2). Enhanced Absorption due to Inhomogeneity

For \( \bar{w}_0 < 1 \), a slightly more involved analysis is required. Eq. (28) describes the effect of non-isotropic scattering on the apparent single scatter albedo of a cloud layer. Substituting \( g' \) for \( g \) allows us to compute the similarity transformed single scatter albedo \( \bar{w}_0 \) as a function of \( \tau' \). For clarity, the quantity \( 1 - \bar{w}_0 \) is plotted in Fig. 9. It is apparent that inhomogeneity is capable of producing a significant increase in the bulk effective single scattering co-albedo of a cloud, which in turn implies enhanced absorption per unit optical depth.

The approximate magnitude of the enhanced absorption effect can be examined analytically using the modified two-stream approximation, as summarized for example by Lenoble (1985). If we employ the similarity-transformed single scatter albedo \( \bar{w}_0 \), then \( \tilde{g} \) becomes zero, in which case the approximate albedo of a semi-infinite cloud layer is given by

\[
G = \frac{1 - s}{1 + s}
\]

where

\[
s = (1 - \bar{w}_0) \frac{1}{2}
\]

The results of this approximation for the albedo of a semi-infinite cloud are depicted in Fig. 10. Because the cloud is semi-infinite and therefore does not transmit any radiation to the lower boundary, the in-cloud absorption is given by \( 1 - G \). The results show the potential for a two-fold or more reduction in \( G \) when \( \tau' \sim 100 \). The absorption enhancement factor is largest in relative terms in the most weakly absorbing case and becomes significantly greater than one even for \( \tau' \) as small as \( \sim 10 \). In terms of the absolute increase in absorption, the enhancement is greatest for the intermediate case of \( \bar{w}_0 = 0.99 \).

5. Comparison to Cairns et al. (2000)

Cairns et al. (2000) derived a theoretical renormalization of the optical properties of inhomogeneous clouds that is identical in purpose, and largely consistent in both results and main conclusions, to the present analysis. The methods, however, are very different, as Cairns’ method is based on a perturbative expansion of the radiative transfer equation, assuming that “the correlation length [in the cloud] is of the same order as the [photon] mean free path and the droplet density fluctuations have lognormal statistics.” Note that the ISC model presented herein makes neither assumption.

Precisely because Cairns’ development is so different from that described herein, it is instructive to identify areas of consistency in the results of both approaches.

Here we consider the simplified correction expressions given by their equation (11):

\[
\sigma' = \sigma(1 - V)^{-1},
\]

Fig. 9. The similarity-transformed effective scattering co-albedo of spherical cloudlets, given by Eq. (28) applied to \( \bar{w}_0(\tau') \) and \( g'(\tau') \), for three values of the intrinsic single scatter albedo \( \bar{w}_0 \) and \( g = 0.86 \).
Two-stream Albedo, Semi-Infinite Cloud, \( g = 0.86 \)

\[
\begin{align*}
G & = 0.999 \\
G & = 0.99 \\
G & = 0.9
\end{align*}
\]

\( \tau' \)

Fig. 10. The approximate albedo of a semi-infinite inhomogeneous cloud, computed from the modified two-stream approximation applied to \( \varpi_0'(\tau') \) and \( g'(\tau') \), for three values of \( \varpi_0 \) and \( g = 0.86 \).

Renormalized Single-Scatter Albedo

\( \varpi_0 \)

G

Fig. 11. Comparison between renormalized single-scatter albedo derived numerically for spherical cloudlets and that derived using the Cairns et al. correction (33). See text for explanation of the mapping between \( \tau' \) and Cairns’ parameter \( V \).

Renormalized Scattering Asymmetry Factor

\( g = 0.86 \)

\[
\begin{align*}
\varpi_0' &= \varpi_0 / [1 + V(1 - \varpi_0)], \quad (33) \\
g' &= g[1 + V(1 - \varpi_0)]/[1 + V(1 - \varpi_0 g)], \quad (34)
\end{align*}
\]

where \( V = \exp(\delta^2) \) and \( \delta \) is the log standard deviation of the cloud water density. Equation (32) has an identical purpose to (21) and (22), but the latter is a function of our \( \tau' \) rather than \( V \). By equating the two expressions, a direct functional mapping from \( \tau' \) to \( V \) is obtained. We may then compute \( \varpi_0' \) and \( g' \) from (33) and (34) and compare these results with the corresponding (numerically computed) ISC values previously plotted in Figs. 5 and 7. Comparisons are shown for selected values of \( \varpi_0 \) and \( g \) in Figs. 11 and 12.

For the renormalized single-scatter albedo (Fig. 11), the agreement between the ISC and Cairns et al. results is excellent for all values of \( \tau' \) less than order 10\(^2\). The implication is that both methods arrived, via radically different theoretical paths, at the same mapping between renormalized optical depth and renormalized single-scatter albedo. According to Cairns et al, their renormalization method is expected to break down for \( \delta > 1 \), which corresponds to \( \tau' > 5 \). Yet the surprisingly good agreement between the two independently derived mappings continues far beyond this threshold.

The comparison between Cairns’ renormalized asymmetry parameter and that derived numerically herein is less satisfactory (Fig. 12). Excellent agreement between the two sets of results is obtained up to \( \tau' \approx 5 \) only for weak absorption and large \( g \). A characteristic
of the Cairns renormalization that is clearly inconsistent, both conceptually and quantitatively, with the ISC model is that their $g'$ cannot be negative, whereas the our numerical results show the expected steady progression from predominantly forward scattering to predominantly backscattering as $\tau'$ increases. The inconsistency is particularly apparent for $g = 0$, where the Cairns' renormalization formula always yields $g' = g = 0$.

In view of the impressive consistency between the Cairns results and the ISC model in at least the case of $\omega_0'$, it appears that there is a strong, if as yet obscure, underlying theoretical link between the two distinct methods. This consistency is encouraging in three respects: 1) It tends to lend credence to both methods; 2) it supports the hypothesis that the geometric details of the small-scale structure of a 3-D inhomogeneous cloud layer are irrelevant to its bulk radiative transfer properties; and 3) it raises the prospect that a simple modification or correction to (34) might be found that generally backscattering as $\tau'$ increases. However, since only three radiative parameters — $\tau^*$, $\omega_0$, and $g$ — are significant for computing fluxes in a layer cloud, it should be possible to adequately model an arbitrary inhomogeneous cloud layer with at most three independent inhomogeneity-related parameters. One candidate would be a cloud layer which is assumed to be composed of two distinct cloudlet types defined by the parameters $(\tau_1', W_1)$ and $(\tau_2', W_2)$, respectively, where $W_i = N_i M_i$ is the contribution of each of the two cloudlet types to the total mean liquid water path $\overline{W}$. Because the latter is a fixed parameter of the problem, $W_1$ and $W_2$ are not independent; rather, a single parameter $f$ may be used to specify the partitioning of liquid water between the two cloudlet types:

$$W_1 = f\overline{W}$$

$$W_2 = (1 - f)\overline{W}$$

A working hypothesis therefore is that the radiative effects of inhomogeneity in a cloud layer with specified mean liquid water path $\overline{W}$ may be parameterized without significant loss of generality in terms of only $\tau_1'$, $\tau_2'$, and $f$. We further conjecture, without theoretical grounds for doing so, that one of the three parameters may be eliminated by fixing $\tau_1' = 0$, in which case the total liquid water content of a cloud is effectively decomposed into a homogeneous and an inhomogeneous component. The two parameters $f$ and (dropping the subscript ‘1’) $\tau'$ then define, respectively, the fraction of the total cloud liquid water which is inhomogeneous and the effective mean optical diameter of the embedded inhomogeneous cloudlets.

Although these simplifications are undertaken without clear theoretical justification, it will be shown both later in this section and in Section 7 that the reduction to two free parameters is empirically defensible. That is, the model is apparently able to reproduce the radiative properties of a wide spectrum of inhomogeneous cloud layers with only the two parameters $f$ and $\tau'$.

With the above simplifications and assumptions, and making use of (11)–(17), the effective optical properties of cloud whose inhomogeneity is characterized by parameters $\tau'$ and $f$ may be summarized as follows:

$$\tau_{\text{eff}}^* = \tau_{\text{hom}} + \tau_{\text{inhom}} \equiv g' \sigma \overline{W}$$
The dependence of the optical depth (or mass extinction) reduction factor $\phi''$ and the effective asymmetry factor $g''$ on the inhomogeneity parameters $\tau'$ and $f$, as defined for a two-parameter non-absorbing inhomogeneous cloud layer.

$$\varpi_0'' = \frac{\tau_{\text{hom}}\varpi_0 + \tau_{\text{inhom}}\varpi_0'}{\tau_{\text{hom}} + \tau_{\text{inhom}}}$$  \hspace{1cm} (38)

$$g'' = \frac{\tau_{\text{hom}}\varpi_0 g + \tau_{\text{inhom}}\varpi_0' g'}{\tau_{\text{hom}} + \tau_{\text{inhom}}}$$  \hspace{1cm} (39)

where

$$\tau_{\text{hom}} = (1 - f)\sigma W$$  \hspace{1cm} (40)

$$\tau_{\text{inhom}} = f\phi(\tau')\sigma W$$  \hspace{1cm} (41)

and the bulk optical depth reduction factor $\phi''$ is defined as

$$\phi'' = \frac{\tau_{\text{eff}}^*}{\sigma W} = [(1 - f) + f\phi(\tau')]$$  \hspace{1cm} (42)

Note that these equations permit not only a two-parameter version of the ISC renormalization model but also a generalization of the Cairns et al. single-parameter renormalization expressions, as their expressions for $\varpi_0'(V)$ and $g'(V)$ may be substituted into (38) and (39), while also substituting $\phi = 1/(1 - V)$ for $\phi(\tau')$ as given by (22).

For the non-absorbing case with $g = 0.86$, Fig. 13 depicts the dependence of $\phi''$ and $g''$ on $\tau'$ and $f$. The envelope encompasses all possible combinations of $f$ and $\tau'$, up to a maximum value of $\tau' = 262.1$. The dashed curve along the lower edge of the envelope corresponds to $f = 1$, for which the cloud consists entirely of discrete cloudlets of optical thickness $\tau'$, as discussed in the previous section. The cusp on the right hand end of the envelope corresponds to small $f$ and/or small $\tau'$; in other words, the limiting homogeneous case. In general, mixing inhomogeneous and homogeneous cloud components is seen to permit larger effective asymmetry factor $g''$ for a given $\tau_{\text{eff}}^*$ than would be possible from a cloud consisting of uniform cloudlets alone. It can further be shown that if $\tau'$ is allowed to become arbitrarily large, the envelope of permissible combinations of $g''$ and optical depth will be exactly bounded at the top by the line $g'' = g = 0.86$, independent of $\phi''$.

The above observation is significant, because it confirms that the present two-parameter inhomogeneity model is sufficient to account for all physically admissible combinations of effective optical depth and effective asymmetry factor in the non-absorbing case. Numerical experiments appear to confirm that an $n$-parameter model that considered a broader range of cloudlet sizes cannot extend the envelope to asymmetry factors greater than $g$, nor can it produce combinations of $g''$ and $\phi''$ falling below the curve defined by the $f = 1$ curve in Fig. 13.

The analysis up to this point considers only the non-absorbing case. For absorbing cases, it is convenient to consider the effect of $\tau'$ and $f$ on the similarity-transformed single scatter albedo and optical depth, derived using (28) and (29) respectively. Envelopes for $\varpi_0$ equal to 0.999, 0.99, and 0.9 are depicted in Fig. 14. Once again, it is apparent that inhomogeneity can...
potentially have a profound effect on the apparent absorptivity of a cloud layer, for constant effective optical depth. In particular, it is clear that significantly different intrinsic values of $\omega_0$ can potentially map to identical bulk cloud optical properties when inhomogeneity is present in varying degrees, suggesting an important potential ambiguity in the interpretation of aircraft or satellite radiometric observations.

7. Validation for Quasi-Realistic Cloud Structures

As emphasized earlier, the geometric foundation of the Independently Scattering Cloudlet model is highly idealized in that it assumes widely separated (non-overlapping) regions of cloudy air separated by clear space. This is analogous to the spatial distribution of individual cloud droplets and is the property that allows the radiative characteristics of individual cloudlets to be summed in a linear fashion. If an actual cloud volume possessed this unusual structure, there is little reason to doubt that the model derived herein should “work” for that case, and indeed this expectation was corroborated empirically with a direct Monte Carlo simulation in Section 4a.

It is much less obvious that the model should have any relevance to clouds possessing a more realistic internal structure, one possibly lacking any clear interior spaces whatsoever. Nevertheless, in view of the highly diffusive properties of radiation, it is reasonable to hypothesize that the area- or volume-averaged radiative properties of any isotropically inhomogeneous cloud volume may be replicated in most important details by the cloudlet model, provided only that the two parameters $f$ and $\tau'$ are chosen correctly (possibly empirically) for that cloud structure. If true, then knowledge of those two parameters alone should be sufficient to calculate a whole spectrum of radiative properties using standard plane-parallel radiative transfer codes.

a. Inhomogeneous cloud model

As an initial test of this hypothesis, Monte Carlo radiative transfer calculations were performed for simulated 3-D cloud structures. The structures were generated by initializing a $32 \times 32 \times 32$ array with Gaussian white noise, Fast-Fourier Transforming the 3-D field into frequency space, multiplying the result by a power-law filter function with negative exponent $b$, inverse FFT-ing the result, and normalizing to zero mean and a user-specified variance $\sigma$. This filtered noise field is intended to simulate a turbulence-generated field of relative humidity perturbations which are then superimposed on a parabolic vertical profile of mean relative humidity. Grid boxes in which the resulting relative humidity is less than 100% are considered cloud free; the remaining grid boxes are assigned a cloud water density in proportion to the excess humidity (Fig. 15). Note that this method resembles that employed by Barker and Davies (1992), except that the present method yields 3-D rather than 2-D cloud distributions.

There are other notable differences between the cloud model employed here and 3-D stochastic models utilized in papers by other authors (Evans, 1993; Cairns et al., 2000). For example, we do not exponentiate the results to obtain a log-normal cloud density distribution. Although some measurements in cloud are suggestive of log-normal density distributions, there is no known reason to expect such a distribution on microphysical or thermodynamic grounds. In particular, a log-normal model precludes both the appearance of cloud-free voids internal to the cloud layer and also precludes a reasonably natural transition to cloud-free air above and below.
Fig. 16. Graphical depiction of simulated 3-D inhomogeneous cloud fields used in test of the ISC model against full Monte Carlo calculations. Left column: Vertically integrated cloud water path. Middle column: Three-dimensional rendering of non-zero liquid water content (LWC). Right column: Histogram of LWC. Fields were generated using power-law filtered white noise (see text), with exponent values of a) $b = -5/3$, b) $b = -2$, c) $b = -7/3$, d) $b = -8/3$, e) $b = -3$. 
the cloud layer.

In fact, cloud-free voids are the expected result when large turbulence- or convection-induced fluctuations in relative humidity (e.g., due to dry-air entrainment at the sides or top of a cloud) are superimposed on layer of only modest average cloud water density. Wherever the total water mixing ratio falls below the saturation mixing ratio, the cloud water evaporates completely. A log-normal model cannot accommodate this case.

In the five validation cases considered here, a constant domain-average optical depth of $\tau^* = 16$ was specified, corresponding to an average liquid water path $W = 0.106$ kg m$^{-2}$ when $r_e = 10$ $\mu$m. As an aid to visualization and to maximize the radiative effects of the inhomogeneity for the purposes of this test, the maximum of the mean humidity profile was chosen such that cloud-free voids occurred rather frequently between regions of denser cloud water.

The variance of local cloud water density was subsequently rescaled so as to be constant for all cases. Thus, the only structural parameter which varied in the simulations was the power-law filter exponent $b$, which determines how the fixed variance in cloud water density is apportioned between large-scale and small-scale structures. Values of $b$ were varied from $-\frac{5}{3}$ to $-3$ in increments of $\frac{1}{3}$. The first of these values was chosen because it approximates the observed spectrum of natural turbulence.

The test cases are depicted graphically in Fig. 16. Note that the large-scale structure is the same in all cases, but that small-scale structure is deemphasized with more negative values of $b$. Note also that the histograms of cloud water density are essentially identical in all five cases. Thus, any differences in the mean radiative properties of these cases can be attributed only to the spatial organization of the cloud water, not the pdf of cloud water density.

For each case, a 3-D Monte Carlo radiative transfer
code was used to compute domain-averaged fluxes for a solar zenith angle of 60°, assuming periodic lateral boundary conditions and a black lower boundary. A Henyey-Greenstein scattering phase function was used, with asymmetry parameter $g = 0.86$. Computations were repeated for intrinsic single scatter albedos $\varpi_0$ of 1.000, 0.999, 0.99, and 0.9. 

The Monte Carlo-computed domain-averaged direct transmittance in each case uniquely determines the effective optical depth $\tau_{\text{eff}}$, via (10), and thus the value of the optical depth reduction factor $\phi(f, \tau')$. In order to determine the appropriate values of $f$ and $\tau'$ for use in the cloudlet model, it is necessary to match one additional domain-averaged property of the cloud layer. The characteristic we arbitrarily chose to match was the domain-averaged diffuse transmittance when $\varpi_0 = 1$.

Once the two parameters $f$ and $\tau'$ are estimated for a given cloud structure (i.e., value of $b$ in these simulations) in the non-absorbing case, all domain-averaged fluxes for both absorbing and non-absorbing cases are readily computed using a plane-parallel radiative transfer code employing the appropriate values of $\tau_{\text{eff}}$, $\varpi_0''$, and $P'$($\theta$). Thus, in effect we are testing the ability of a simple two-parameter model to correctly predict the functional dependence of three independent variables — the domain-averaged reflectance, diffuse transmittance, and in-cloud absorptance — on $\varpi_0$.

Table 2 gives both the values of the fitted parameters $\tau'$ and $f$ and the results of the ISC-based plane-parallel radiative transfer calculations and for the commonly used Independent Pixel Approximation (IPA). The latter is derived by computing local fluxes from the local optical thickness $\tau^*$ and then averaging over the domain.

For Case 1, which has the most fine scale structure, $\tau' \approx 12$ and $f \approx 0.93$. Case 5, which in many respects resembles the actual cloud field depicted in Fig. 1, yields $\tau' \approx 21$ and $f = 1$. The large value of $f$ determined in all five cases is consistent with the relatively high degree of delineation of the denser cloud elements, as contrasted with a situation in which they were embedded within more or less continuous cloud.

In all five cases, the variables derived from ISC are, overall, in markedly better agreement with the direct MC calculations than are the IPA results. ISC results printed in bold were not constrained to agree with the MC results; therefore these results serve as the primary basis for evaluating the ISC model.

Only in the strongly absorbing cases ($\varpi_0 = 0.9$) were the results mixed. In these cases, the ISC tended to overestimate absorption to a moderate degree, with compensating underestimates of reflection and diffuse transmittance. By contrast, the IPA results tend to strongly underestimate absorption, with compensating overestimates of reflection and direct transmittance. The IPA yields either overestimates or underestimates of the diffuse transmittance, depending on both $b$ and $\varpi_0$.

It is important to note that the method chosen for determining $f$ and $\tau'$ in this comparison is arbitrary and may have some bearing on the size of the errors encountered in the ISC results. One proposed alternative is to choose these parameters so as to match spherical albedos and transmittances, rather than fluxes calculated for a single incident sun angle of 60°. This of course will require a much larger set of direct Monte Carlo simulations for the comparison data set.

8. Conclusions

A novel, yet conceptually and computationally straightforward stochastic model has been proposed for treating a three-dimensionally inhomogenous cloud layer (or volume) as a radiatively equivalent homogeneous layer (or volume) having modified optical depth $\tau_{\text{eff}}$, single scatter albedo $\varpi_0''$, and asymmetry parameter $g''$. Although the renormalization of optical properties of clouds has been studied by previous authors, this is believed to be the first based on a model in which macroscopic cloud structures and/or embedded inhomogeneities function as discrete, independent scatterers for radiative transfer purposes.

It was argued, and to a limited extent empirically demonstrated, that as few as two independent structural parameters $f$ and $\tau'$ appear to be adequate to capture the most important radiative effects of random internal inhomogeneities, such as those produced by turbulent processes within clouds. Both of these parameters are dimensionless quantities that have nothing to do with spatial scales of variability or the amplitude of local cloud density fluctuations but rather give a measure of the optical “lumpiness” of the cloud volume. The establishment of a theoretical link between these parameters and those commonly reported in aircraft measurements remains a topic for further study.

Although both the method of analysis and the underlying assumptions behind the present Independently Scattering Cloudlet model are quite different from those employed in the renormalization formulation of Cairns et al. (2000), remarkable consistency was noted between the two methods as regards the relationship be-
Table 2. Comparison between Independently Scattering Cloudlet (ISC) model, Monte Carlo (MC), and Independent Pixel Approximation (IPA) results for the cases depicted in Fig. 16. ISC model parameters \( f \) and \( \tau' \), which determine \( \tau^*_{\text{eff}} \), \( \varpi_0' \), and \( g' \) used in the plane-parallel radiative transfer code, were themselves determined by reference to MC results for case that \( \varpi_0 = 1 \) (see text). ISC results in boldface are those which were not constrained to agree with the MC results.

### Case 1: \( b = -\frac{5}{3} \)  \( \tau^* = 16 \)  \( g = 0.86 \)  \( \mu_0 = 0.5 \)
Fitted model parameters: \( f = 0.935 \)  \( \tau' = 11.81 \)
Renormalized optical depth: \( \tau^*_{\text{eff}} = 3.024 \)

<table>
<thead>
<tr>
<th>( \varpi_0 )</th>
<th>( \varpi_0'' )</th>
<th>( g'' )</th>
<th>Reflect.</th>
<th>Dir. Trans.</th>
<th>Dif. Trans.</th>
<th>Tot. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.547</td>
<td>ISC</td>
<td>0.580</td>
<td>0.002</td>
<td>0.418</td>
<td>0.420</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.580</td>
<td>0.002</td>
<td>0.418</td>
<td>0.420</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.567</td>
<td>0.067</td>
<td>0.366</td>
<td>0.433</td>
</tr>
<tr>
<td>0.999</td>
<td>0.995</td>
<td>0.550</td>
<td>ISC</td>
<td>0.561</td>
<td>0.002</td>
<td>0.404</td>
<td>0.406</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.565</td>
<td>0.002</td>
<td>0.403</td>
<td>0.406</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.550</td>
<td>0.067</td>
<td>0.355</td>
<td>0.422</td>
</tr>
<tr>
<td>0.99</td>
<td>0.950</td>
<td>0.560</td>
<td>ISC</td>
<td>0.439</td>
<td>0.002</td>
<td>0.302</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.443</td>
<td>0.002</td>
<td>0.302</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.454</td>
<td>0.067</td>
<td>0.283</td>
<td>0.350</td>
</tr>
<tr>
<td>0.9</td>
<td>0.652</td>
<td>0.641</td>
<td>ISC</td>
<td>0.118</td>
<td>0.002</td>
<td>0.071</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.130</td>
<td>0.002</td>
<td>0.074</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.178</td>
<td>0.067</td>
<td>0.105</td>
<td>0.172</td>
</tr>
</tbody>
</table>

### Case 2: \( b = -2 \)  \( \tau^* = 16 \)  \( g = 0.86 \)  \( \mu_0 = 0.5 \)
Fitted model parameters: \( f = 0.959 \)  \( \tau' = 14.80 \)
Renormalized optical depth: \( \tau^*_{\text{eff}} = 2.255 \)

<table>
<thead>
<tr>
<th>( \varpi_0 )</th>
<th>( \varpi_0'' )</th>
<th>( g'' )</th>
<th>Reflect.</th>
<th>Dir. Trans.</th>
<th>Dif. Trans.</th>
<th>Tot. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.459</td>
<td>ISC</td>
<td>0.553</td>
<td>0.011</td>
<td>0.436</td>
<td>0.447</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.553</td>
<td>0.011</td>
<td>0.436</td>
<td>0.447</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.530</td>
<td>0.112</td>
<td>0.358</td>
<td>0.470</td>
</tr>
<tr>
<td>0.999</td>
<td>0.993</td>
<td>0.462</td>
<td>ISC</td>
<td>0.535</td>
<td>0.011</td>
<td>0.421</td>
<td>0.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.537</td>
<td>0.011</td>
<td>0.421</td>
<td>0.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.514</td>
<td>0.112</td>
<td>0.346</td>
<td>0.458</td>
</tr>
<tr>
<td>0.99</td>
<td>0.934</td>
<td>0.475</td>
<td>ISC</td>
<td>0.412</td>
<td>0.011</td>
<td>0.317</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.415</td>
<td>0.011</td>
<td>0.320</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.423</td>
<td>0.112</td>
<td>0.281</td>
<td>0.393</td>
</tr>
<tr>
<td>0.9</td>
<td>0.582</td>
<td>0.582</td>
<td>ISC</td>
<td>0.104</td>
<td>0.011</td>
<td>0.085</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.119</td>
<td>0.011</td>
<td>0.097</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.167</td>
<td>0.112</td>
<td>0.118</td>
<td>0.230</td>
</tr>
</tbody>
</table>
Table 2. (cont.)

**Case 3:** \( b = -7/3 \) \( \tau^* = 16 \) \( g = 0.86 \) \( \mu_0 = 0.5 \)
Fitted model parameters: \( f = 0.977 \) \( \tau' = 17.40 \)
Renormalized optical depth: \( \tau^*_{\text{eff}} = 1.715 \)

<table>
<thead>
<tr>
<th>( \omega_0 )</th>
<th>( \omega_0'' )</th>
<th>( g'' )</th>
<th>Reflect.</th>
<th>Dir. Trans.</th>
<th>Diff. Trans.</th>
<th>Tot. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.359</td>
<td>ISC 0.529</td>
<td>0.032</td>
<td>0.439</td>
<td>0.471</td>
<td>0.000</td>
</tr>
<tr>
<td>MC 0.529</td>
<td>0.032</td>
<td>0.439</td>
<td>0.471</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.491</td>
<td>0.181</td>
<td>0.328</td>
<td>0.509</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.999</td>
<td>0.991</td>
<td>0.361</td>
<td>ISC 0.511</td>
<td>0.032</td>
<td>0.424</td>
<td>0.456</td>
<td>0.033</td>
</tr>
<tr>
<td>MC 0.511</td>
<td>0.032</td>
<td>0.425</td>
<td>0.457</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.475</td>
<td>0.180</td>
<td>0.318</td>
<td>0.498</td>
<td>0.027</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>0.914</td>
<td>0.376</td>
<td>ISC 0.388</td>
<td>0.032</td>
<td>0.319</td>
<td>0.351</td>
<td>0.261</td>
</tr>
<tr>
<td>MC 0.391</td>
<td>0.032</td>
<td>0.325</td>
<td>0.357</td>
<td>0.252</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.388</td>
<td>0.180</td>
<td>0.258</td>
<td>0.438</td>
<td>0.174</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.504</td>
<td>0.500</td>
<td>ISC 0.093</td>
<td>0.032</td>
<td>0.090</td>
<td>0.122</td>
<td>0.785</td>
</tr>
<tr>
<td>MC 0.110</td>
<td>0.032</td>
<td>0.112</td>
<td>0.144</td>
<td>0.746</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.153</td>
<td>0.180</td>
<td>0.115</td>
<td>0.295</td>
<td>0.552</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. (cont.)

**Case 4:** \( b = -8/3 \) \( \tau^* = 16 \) \( g = 0.86 \) \( \mu_0 = 0.5 \)
Fitted model parameters: \( f = 0.997 \) \( \tau' = 19.72 \)
Renormalized optical depth: \( \tau^*_{\text{eff}} = 1.306 \)

<table>
<thead>
<tr>
<th>( \omega_0 )</th>
<th>( \omega_0'' )</th>
<th>( g'' )</th>
<th>Reflect.</th>
<th>Dir. Trans.</th>
<th>Diff. Trans.</th>
<th>Tot. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.227</td>
<td>ISC 0.505</td>
<td>0.073</td>
<td>0.422</td>
<td>0.495</td>
<td>0.000</td>
</tr>
<tr>
<td>MC 0.505</td>
<td>0.073</td>
<td>0.422</td>
<td>0.495</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.455</td>
<td>0.257</td>
<td>0.288</td>
<td>0.545</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.999</td>
<td>0.988</td>
<td>0.228</td>
<td>ISC 0.486</td>
<td>0.073</td>
<td>0.407</td>
<td>0.480</td>
<td>0.034</td>
</tr>
<tr>
<td>MC 0.488</td>
<td>0.073</td>
<td>0.407</td>
<td>0.480</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.438</td>
<td>0.257</td>
<td>0.278</td>
<td>0.535</td>
<td>0.027</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td>0.889</td>
<td>0.239</td>
<td>ISC 0.365</td>
<td>0.073</td>
<td>0.303</td>
<td>0.376</td>
<td>0.259</td>
</tr>
<tr>
<td>MC 0.368</td>
<td>0.073</td>
<td>0.311</td>
<td>0.384</td>
<td>0.248</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.353</td>
<td>0.258</td>
<td>0.224</td>
<td>0.482</td>
<td>0.165</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.404</td>
<td>0.271</td>
<td>ISC 0.084</td>
<td>0.073</td>
<td>0.081</td>
<td>0.154</td>
<td>0.761</td>
</tr>
<tr>
<td>MC 0.104</td>
<td>0.073</td>
<td>0.114</td>
<td>0.187</td>
<td>0.709</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPA 0.138</td>
<td>0.257</td>
<td>0.103</td>
<td>0.360</td>
<td>0.502</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


**Table 2. (cont.)**

<table>
<thead>
<tr>
<th>$\varpi_0$</th>
<th>$\varpi_0^\prime$</th>
<th>$\varpi'$</th>
<th>Reflect.</th>
<th>Dir. Trans.</th>
<th>Diff. Trans.</th>
<th>Tot. Trans.</th>
<th>Absorp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>0.154</td>
<td>ISC</td>
<td>0.488</td>
<td>0.120</td>
<td>0.392</td>
<td>0.512</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.488</td>
<td>0.120</td>
<td>0.392</td>
<td>0.512</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.428</td>
<td>0.321</td>
<td>0.251</td>
<td>0.572</td>
</tr>
<tr>
<td>0.999</td>
<td>0.985</td>
<td>0.155</td>
<td>ISC</td>
<td>0.470</td>
<td>0.120</td>
<td>0.376</td>
<td>0.496</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.470</td>
<td>0.120</td>
<td>0.377</td>
<td>0.497</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.412</td>
<td>0.321</td>
<td>0.241</td>
<td>0.562</td>
</tr>
<tr>
<td>0.99</td>
<td>0.865</td>
<td>0.161</td>
<td>ISC</td>
<td>0.349</td>
<td>0.120</td>
<td>0.273</td>
<td>0.393</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.353</td>
<td>0.120</td>
<td>0.285</td>
<td>0.405</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.331</td>
<td>0.321</td>
<td>0.190</td>
<td>0.511</td>
</tr>
<tr>
<td>0.9</td>
<td>0.309</td>
<td>0.227</td>
<td>ISC</td>
<td>0.078</td>
<td>0.120</td>
<td>0.059</td>
<td>0.179</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>MC</td>
<td>0.101</td>
<td>0.120</td>
<td>0.104</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IPA</td>
<td>0.127</td>
<td>0.321</td>
<td>0.085</td>
<td>0.406</td>
</tr>
</tbody>
</table>

between $\tau_{\text{eff}}^\ast$ and $\varpi'$'. This agreement suggests that both methods may be independently converging on a quasi-universal principle of renormalization in 3-D inhomogeneous clouds, one that perhaps does not depend on the detailed geometric or statistical assumptions employed. An inconsistency between the ISC and the Cairns model is noted, however, in the case of the renormalized scattering asymmetry parameter. We tentatively attribute the disagreement to a potentially correctable deficiency in the Cairns formulation for $g'$. If a revised analytical expression for $g'$ could be found that agrees more closely with our numerical results, the application of the ISC model to arbitrary cloud fields would not only be greatly simplified, but also would be established between our parameter $\tau'$, which describes the effective mean optical diameter of embedded inhomogeneities, and the log standard deviation $\delta$ of the cloud density distribution.

The Cairns formulation is essentially a one-parameter model, depending only on $\delta$. Our two-parameter ISC model yields a less restrictive relationship between $\tau_{\text{eff}}^\ast$, $\varpi'$, and $g'$ and therefore may be applicable to a broader spectrum of cloud structures. Note, however, that our relationships (37)–(42) may also be used to generalize the Cairns model to two parameters.

It must be reiterated that the mathematical derivation of the new model was based on a highly idealized model of the geometric structure of an inhomogeneous cloud volume. Our key working hypothesis, therefore, was that the domain-averaged effects of inhomogeneity on radiative transfer in actual clouds are actually insensitive to the details of the internal cloud structure, provided only that the model parameters are chosen correctly. Empirical comparisons with direct Monte Carlo radiative transfer calculations for one family of quasi-realistic cloud structures appear to strongly support this hypothesis.

Considerable additional validation of the above type is needed in order to establish the full range of applicability of the ISC model to realistic cloud structures. Among other things, it may be preferable to choose model parameters so as to match spherical albedo and transmitted fluxes, rather than matching flux values for a single solar incidence angle.

Also, because most clouds in nature tend to exhibit different scales of variability in the horizontal and vertical, we speculate that such clouds will be found to be best represented by a hybrid of the ISC and the IPA methods. An alternative possibility would be to generalize the ISC model to include oriented, non-spherical cloudlets, which would be computationally analogous to plane-parallel radiative transfer calculations for clouds composed of oriented non-spherical particles. Thus, in the azimuthally isotropic case, renormalized single-scatter properties for an embedded cloudlet could depend on the radiation incidence angle $\theta$.

All of these possibilities may be explored via suitable Monte Carlo simulations as well as through analysis of
actual field measurements. The latter could for example include spatially (or temporally) averaged direct and diffuse transmittance and/or albedo at selected wavelengths for which gaseous absorption is negligible but for which the intrinsic single-scatter albedo of the cloud layer can be estimated with reasonable accuracy.

If the ISC model is indeed found to adequately reproduce the average radiative properties for many real cloud structures, then it may offer a computationally simple yet physically self-consistent basis for improving parameterizations of solar radiative transfer in clouds, such as those required in general circulation models (GCMs).

Finally, it must be emphasized that the ISC model is not intended as a substitute for rigorous theories of radiative transfer in inhomogeneous clouds. Rather, its intended role is analogous to that of the so-called “eddy viscosity” model widely used to parameterize momentum transport in the turbulent boundary layer, based on the concept of an equivalent laminar flow with modified viscosity. What the latter model lacks in theoretical rigor it more than makes up for by 1) yielding numerical results that are superior to those obtained using the far more physically grounded, yet clearly inapplicable, “intrinsic” molecular viscosity and 2) requiring vastly less computational effort than high-resolution simulations of actual turbulence. Both qualities are essential within the framework of numerical climate and weather prediction models. Moreover, the eddy viscosity model of turbulent transfer is pedagogically useful in that it permits the net consequences of a highly complex process to be understood at a reasonably intuitive level. The present ISC model for stochastic radiative transfer is developed with the same class of applications in mind and with similar caveats concerning its validity as a literal model of reality.

Acknowledgments The encouraging responses of Alexander Marshak, Anthony Davis, Stephen Warren, and James Weinman to early presentations of the ISC model provided the impetus to see this paper through to publication. Comments from Warren Wiscombe, Robert Cahalan, and anonymous reviewers prompted a strengthening of the validation component of this study and many other important revisions. Brian Cairns and Michael Hobson brought to the author’s attention several relevant papers in the astrophysical literature. Jeff Key and Bryan Baum provided additional helpful comments. This study was undertaken while the author was funded by NASA Grants NAG5-9894 and NAG5-9999.

References


